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The Effects of Non-Local Operators in Rare B Decays, $B \to X_s l^+ l^-$

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Abstract

The effects of non-local operators in rare B decays, $B \to X_s l^+ l^-$, are investigated through the model independent analysis. It deals with complete twelve four-Fermi interactions, including two non-local ones which are constrained by the experimental result of the $B \to X_s \gamma$ decay. The branching ratio and the forward-backward asymmetry are studied as functions of the twelve Wilson coefficients. We also show the correlation between the branching ratio and the forward-backward asymmetry via two coefficients of the non-local operators. This will certainly help us find any deviations from the standard model through the non-local interactions.

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I Introduction

Flavor changing neutral current (FCNC) processes are possibly the most sensitive to the various theoretical extensions of the standard model (SM) because these decays occur in the SM only through loops. Non-standard model effects can manifest themselves in these rare decays through the Wilson coefficients, which can have values distinctly different from their standard model counterparts. (See, for example, [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] for the model dependent analysis.) Previously we gave the model-independent analysis on rare B meson decays $B \to X_s l^+ l^-$ in Refs. [11, 12], where we dealt with all the possible local interactions and investigated the property of these interactions. However, we did not include the non-local interactions, just for simplicity. These non-local interactions influence the process $B \to X_s \gamma$. This process is a kind of "rare" decay with quite large branching ratio, of order of 10^{-4} [13], and it has been studied extensively in Refs. [14, 15, 16, 17]. Compared to $B \to X_s \gamma$, the decay $B \to X_s l^+ l^-$ is much more sensitive to the actual form of the new interactions since we can measure experimentally various kinematical distributions as well as the total rate. While new physics can change only the systematically uncertain normalization for $B \to X_s \gamma$, the interplay of various operators will change the spectra of the decay $B \to X_s l^+ l^-$. *E.g.*, the experimental observation of $B \to X_s \gamma$ restricts the absolute value of the Wilson coefficient C_7 of the non-local operator O_7, however, we cannot determine the sign of the C_7 from the decay rate of $B \to X_s \gamma$. But, if we analyze the interference between the non-local interactions and the other operators in the process $B \to X_s l^+ l^-$, we can extract much more information about C_7. Therefore, to search for new physics, it would be most interesting to have a model-independent study for the non-local interactions in $B \to X_s l^+ l^-$ decays.

Here we consider the branching ratio and the forward-backward (FB) asymmetry of inclusive $B \to X_s e^+ e^-$ or $B \to X_s \mu^+ \mu^-$ decay, which are functions of the twelve Wilson coefficients of four-Fermi interactions. The corresponding matrix elements [11, 12] are given as

$$
\begin{aligned}
\mathcal{M}(B \to X_s l^+ l^-) \;=\;& \frac{G_F\ \alpha}{\sqrt{2}\pi}\ V_{ts}^* V_{tb} \times \\
\Big[\;& C_{SL}\ \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} (m_s L) b\ \bar{l} \gamma^\mu l + C_{BR}\ \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} (m_b R) b\ \bar{l} \gamma^\mu l \\
+\;& C_{LL}\ \bar{s}_L \gamma_\mu b_L\ \bar{l}_L \gamma^\mu l_L + C_{LR}\ \bar{s}_L \gamma_\mu b_L\ \bar{l}_R \gamma^\mu l_R \\
+\;& C_{RL}\ \bar{s}_R \gamma_\mu b_R\ \bar{l}_L \gamma^\mu l_L + C_{RR}\ \bar{s}_R \gamma_\mu b_R\ \bar{l}_R \gamma^\mu l_R
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad C_{LRLR}\ \bar{s}_L b_R\ \bar{l}_L l_R + C_{RLLR}\ \bar{s}_R b_L\ \bar{l}_L l_R \\
&+ \quad C_{LRRL}\ \bar{s}_L b_R\ \bar{l}_R l_L + C_{RLRL}\ \bar{s}_R b_L\ \bar{l}_R l_L \\
&+ \quad C_T\ \bar{s}\sigma_{\mu\nu} b\ \bar{l}\sigma^{\mu\nu} l + i C_{TE}\ \bar{s}\sigma_{\mu\nu} b\ \bar{l}\sigma_{\alpha\beta} l\ \epsilon^{\mu\nu\alpha\beta}].
\end{aligned} \tag{1.1}
$$

Here, we represent the Wilson coefficients as C_{XX}'s. The C_{SL} and C_{BR} correspond to the non-local four-Fermi operators, and the other ten coefficients to the local operators. We choose the mass of b-quark, $m_b = 4.8$ GeV, as the renormalization scale μ. The subscripts, L and R, express chiral projection operators, $L = \frac{1}{2}(1-\gamma_5)$ and $R = \frac{1}{2}(1+\gamma_5)$, and thus correspond to the chirality of quark and lepton operators. Thus, there are two non-local operators, C_{SL} and C_{BR} and ten local ones, *i.e.*, four vector-type interactions C_{LL}, C_{LR}, C_{RL} and C_{RR}, four scalar-type ones C_{LRLR}, C_{LRRL}, C_{RLLR} and C_{RLRL}, and two tensor-type ones C_T and C_{TE}. We note that two coefficients of the non-local operators are also constrained by the experimental data of $B \to X_s\gamma$, which will be shown in Sec. II. The SM predicts that:

- Both of the C_{SL} and C_{BR} are equal to $-2C_7^{eff}$, *i.e.*,

$$
m_s^2 \left|C_{SL}\right|^2 + m_b^2 \left|C_{BR}\right|^2 = 4C_7^2(m_b^2 + m_s^2) \approx 4C_7^2 m_b^2. \tag{1.2}
$$

- The C_{LL} and C_{LR} in vector parts are given in terms of C_9^{eff} and C_{10}, that is,

$$
C_{LL} = C_9^{eff} - C_{10} \qquad \text{and} \qquad C_{LR} = C_9^{eff} + C_{10}. \tag{1.3}
$$

- The other coefficients are all negligible, and $\mathcal{M}(B \to X_s l^+ l^-)_{\rm SM}$ becomes

$$
\begin{aligned}
\mathcal{M}(B \to X_s l^+ l^-)_{\rm SM} \ \approx\ & \frac{G_F\alpha}{\sqrt{2}\pi} V_{ts}^* V_{tb} \times \\
& [\quad -2C_7^{eff} \bar{s} i\sigma_{\mu\nu} \frac{q^\nu}{q^2} \left(m_s L + m_b R\right) b \bar{l}\gamma^\mu l \\
& + \quad \left(C_9^{eff} - C_{10}\right) \bar{s}_L \gamma_\mu b_L \bar{l}_L \gamma^\mu l_L \\
& + \quad \left(C_9^{eff} + C_{10}\right) \bar{s}_L \gamma_\mu b_L \bar{l}_R \gamma^\mu l_R].
\end{aligned} \tag{1.4}
$$

- The three coefficients in the SM have been well studied [18, 19], and we set [20, 21, 22, 23]

$$
(C_7, C_9^{NDR}, C_{10}) = (-0.311, 4.153, -4.546).
$$

The paper is organized as follows. In Sec. II, we study the effects due to the non-local interactions on the branching ratio and the FB asymmetry, which are derived from the most general effective Hamiltonian. In Sec. III, we give the correlation between the branching ratio and the FB asymmetry. Conclusions are also in Sec. III.

II Branching Ratio and Forward-Backward Asymmetry of the Process, $B \to X_s l^+ l^-$

We calculate the branching ratio and the forward-backward (FB) asymmetry of the $B \to X_s l^+ l^-$ decay due to the new operators of the models beyond the SM, following the method [11, 12]. We first concentrate on the $B \to X_s \gamma$ decay to get the present constraints on the non-local Wilson coefficients of the $B \to X_s l^+ l^-$ decay. The effective Hamiltonian for the $B \to X_s \gamma$ is given as

$$\mathcal{M}(B \to X_s \gamma) = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i^8 C_i(\mu) \mathcal{O}_i(\mu), \tag{2.1}$$

where C_i's and $\mathcal{O}_i$'s are the relevant Wilson coefficients and the corresponding operators. We show only the $\mathcal{O}_7$ explicitly, that is,

$$\mathcal{O}_7 = \frac{e}{16\pi^2}\ \bar{s}\sigma_{\mu\nu}(m_b\ R + m_s\ L) b\ F^{\mu\nu}, \tag{2.2}$$

where e and $F^{\mu\nu}$ are the electromagnetic coupling constant and the electromagnetic field strength. The resultant branching ratio in the leading order is given as

$$\mathcal{B}(B \to X_s \gamma) = \mathcal{B}_0 \frac{32\pi}{\alpha}\ \left| C_7^{eff} \right|^2, \tag{2.3}$$

where $\mathcal{B}_0$ is the normalization factor, normalized to the semi-leptonic branching fraction $\mathcal{B}_{sl}(B \to X l \nu)$ as

$$\mathcal{B}_0 = \mathcal{B}_{sl}\ \frac{3\alpha^2}{16\pi^2}\ \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2}\ \frac{1}{f(\hat{m}_c)\kappa(\hat{m}_c)}. \tag{2.4}$$

Here $f(\hat{m}_c = \frac{m_c}{m_b})$ and $\kappa(\hat{m}_c)$ are phase space factor and the $\mathcal{O}(\alpha_s)$ QCD correction factor [24] of a process $b \to c l \nu$ given by

$$\begin{aligned} f(\hat{m}_c) &= 1 - 8\hat{m}_c{}^2 + 8\hat{m}_c{}^6 - \hat{m}_c{}^8 - 24\hat{m}_c{}^4 \ln \hat{m}_c, & (2.5) \\ \kappa(\hat{m}_c) &= 1 - \frac{2\alpha_s(m_b)}{3\pi}\ \left[\left(\pi^2 - \frac{31}{4} \right) (1 - \hat{m}_c)^2 + \frac{3}{2} \right]. & (2.6) \end{aligned}$$

For the numerical analysis, we set $\frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} = 1$ and use $\mathcal{B}_{sl} = 10.4\%$, the experimental value of semileptonic branching fraction of $B \to X l \nu$. By measuring the branching fraction of

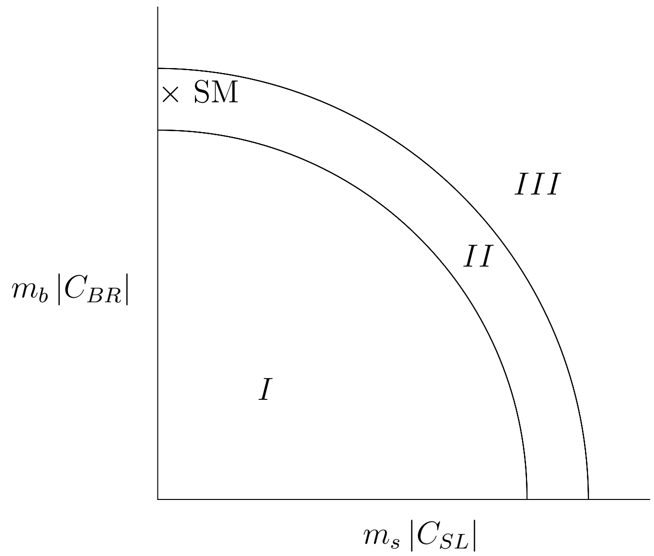

Figure 1: Constraint on C_{SL} and C_{BR} by rare B decays $B \to X_s\gamma$. These coefficients can have values only within the region II. The mark $\times$ denotes the standard model point.

$B \to X_s\gamma$, we can find present constraints on the theory describing the decay $B \to X_s l^+ l^-$, especially on C_7, which also appears as the coefficient of the non-local operators of the decay $B \to X_s l^+ l^-$ [2].

Based on the experimental values of the decay width of $B \to X_s\gamma$, which is consistent with the value of C_7 predicted by the SM [13, 15] as appearing in Eq. (1.2),

$$4C_7^2(m_b^2 + m_s^2) = m_s^2\,|C_{SL}|^2 + m_b^2\,|C_{BR}|^2\,, \tag{2.7}$$

we can easily find that the coefficients of two non-local operators are placed between two fans whose radii are about $2(m_b^2 + m_s^2)^{1/2}|C_7|$ in $(m_s\,|C_{SL}|\,, m_b\,|C_{BR}|)$ plain. And the recent result at CLEO for the branching ratio of the $B \to X_s\gamma$ [13],

$$2.0 \times 10^{-4} < \mathcal{B}(B \to X_s\gamma) < 4.5 \times 10^{-4}\ (95\%\ CL),$$

gives the constraint on the absolute value of $C_7^{eff}(m_b)$ [25], that is,

$$0.28 < |C_7^{eff}(m_b)| < 0.41.$$

As shown in Figure 1, only the values of the coefficients C_{BR} and C_{SL} within the region II can be permitted. Because s-quark mass is much less than b-quark mass, $m_s \ll m_b$, we may regard that the term C_{SL}, which is proportional to m_s, hardly contributes to the $B \to X_s l^+ l^-$ decay in the SM. This means that the SM point is placed near the $m_b\,|C_{BR}|$

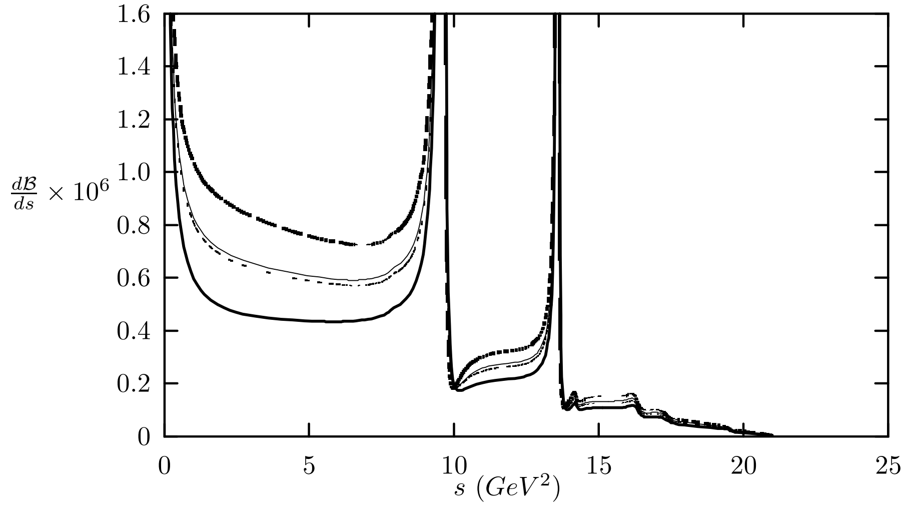

Figure 2: Branching ratio for $(C_{SL}^N, C_{BR}) = (0, -2C_7)$ (thick solid line), $(0, 2C_7)$ (thick dashed line), $(-2C_7, 0)$ (thin solid line) and $(2C_7, 0)$ (dotted line). The thick solid line also represents the branching ratio for the SM.

axis in the $(m_s\,|C_{SL}|\,, m_b\,|C_{BR}|)$ plain, as shown in Figure 1. Assuming that there is no new phase from the non-local interactions, Eq. (2.7) gives, as $m_s \to 0$,

$$-2C_7 \le C_{BR} \le 2C_7, \tag{2.8}$$

and

$$-2C_7 \le C_{SL}^N \le 2C_7, \quad \text{where} \quad C_{SL}^N \equiv \frac{m_s}{m_b} C_{SL}. \tag{2.9}$$

Here, we denoted the normalized C_{SL} as C_{SL}^N. Therefore, it is very important to know the branching ratio at 4 points

$$(C_{SL}^N,\ C_{BR}) = (-2C_7, 0), \quad (2C_7, 0), \quad (0, -2C_7), \quad (0, 2C_7).$$

We show the branching ratio of $B \to X_s l^+ l^-$ for massless lepton case in Figure 2 in the absence of any new local interactions, but with new non-local interactions and the already existing local operators of the SM. In this case, the branching ratio is given as

$$\begin{aligned} \frac{d\mathcal{B}}{ds}(B \to X_s l^+ l^-) &= \frac{1}{2m_b^8}\mathcal{B}_0[S_1(s)m_b^2\{\left|C_{SL}^N\right|^2 + |C_{BR}|^2\} + 2S_2(s)m_b^2 Re[C_{SL}^N C_{BR}^*] \\ &\quad + 4S_3(s)m_b m_s Re[C_{SL}^N C_9^{eff*}] + 4S_4(s)m_b^2 Re[C_{BR} C_9^{eff*}] \\ &\quad + M_2(s)\{\left|C_9^{eff} - C_{10}\right|^2 + \left|C_9^{eff} + C_{10}\right|^2\}], \end{aligned} \tag{2.10}$$

with $s = (p_{l^+} + p_{l^-})^2$, invariant mass-square of lepton pair. (The case with the all twelve operators is given in Apendix.) The $S_n(s)$ and $M_2(s)$ are given in Refs. [11, 12]. In

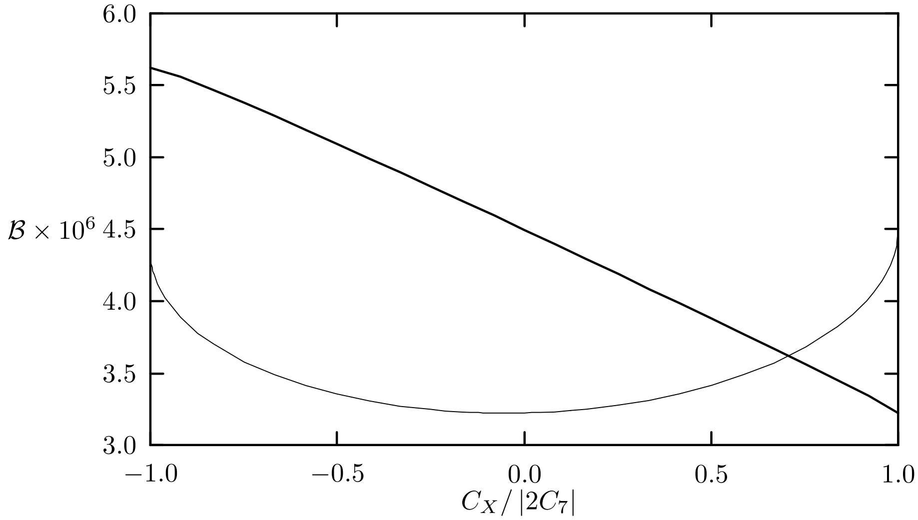

Figure 3: Partially integrated branching ratio $\mathcal{B} \equiv \int_1^8 ds \frac{d\mathcal{B}}{ds}$. The coefficients C_{SL}^N (thin line) and C_{BR} (thick line) move under the conditions (2.8) and (2.9).

Appendix, we show the behavior of the coefficient functions $S_n(s)$. We note that the branching ratio is more sensitive to the change of C_{BR} than of C_{SL}, as shown in Figure 2, because the interferences of the C_{SL} and the C_{BR} to the vector type interactions of the SM give

$$\begin{aligned} & Tr\{\bar{s}\sigma_{\mu\nu} L b (\bar{s}_L \gamma_\rho b_L)^*\} \propto m_s, \\ \text{and} \quad & Tr\{\bar{s}\sigma_{\mu\nu} R b (\bar{s}_L \gamma_\rho b_L)^*\} \propto m_b, \end{aligned}$$

respectively. The contribution from these coefficients oscillates as we vary the values of $(C_{SL}^N,\ C_{BR})$ within the region II in Figure 1, because of the constraint (2.7). However, if we account only for the C_{BR} (as in the SM), or equivalently, if we assume $m_s = 0$, then C_{BR} moves only between $-2\,|C_7|$ and $2\,|C_7|$, and the branching ratio decreases monotonously. The points $(C_{SL}^N, C_{BR}) = (0, 2\,|C_7|)$ and $(0, -2\,|C_7|)$ are the minimum and the maximum values. (The SM value corresponds almost to the point $(0, -2C_7)$.) This behavior reappears for the partially integrated branching ratio $\mathcal{B} \equiv \int_1^8 ds \frac{d\mathcal{B}}{ds}$ [11] as shown in Figure 3.

Now we consider the forward-backward (FB) asymmetry defined as

$$\frac{d\bar{\mathcal{A}}}{ds} \equiv \frac{d\mathcal{A}/ds}{d\mathcal{B}/ds} = \frac{\int_0^1 dz \frac{d^2\mathcal{B}}{dsdz} - \int_{-1}^0 dz \frac{d^2\mathcal{B}}{dsdz}}{\int_0^1 dz \frac{d^2\mathcal{B}}{dsdz} + \int_{-1}^0 dz \frac{d^2\mathcal{B}}{dsdz}}, \tag{2.11}$$

where z is the cosine value of the angle between the momentum of B meson and that of l^+ in the laboratory frame. We show the normalized FB asymmetry curves, $d\bar{\mathcal{A}}/ds$, at

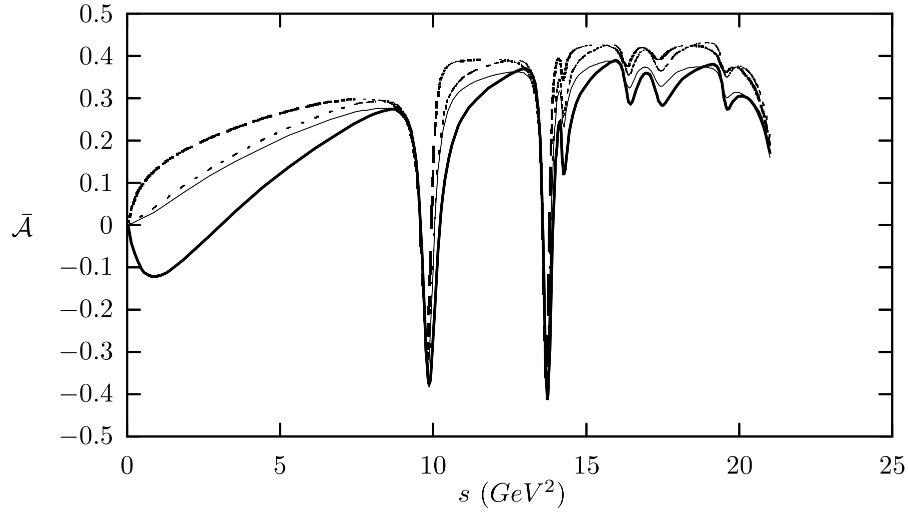

Figure 4: FB asymmetry for $(C_{SL}^N, C_{BR}) = (0, -2C_7)$ (thick solid line), $(0, 2C_7)$ (thick dashed line), $(-2C_7, 0)$ (thin solid line) and $(2C_7, 0)$ (thin dashed lines). The SM value corresponds almost to the point $(0, -2C_7)$.

four points

$$(C_{SL}^N, C_{BR}) = (0, -2C_7), \quad (0, 2C_7), \quad (-2C_7, 0), \quad (2C_7, 0)$$

in Figure 4. The line for the $(0, -2C_7)$ corresponds to the SM result. The unnormalized FB asymmetry of $B \to X_s l^+ l^-$ for massless lepton case in the absence of any new local interactions, but with new non-local operators and the already existing local operators of the SM is given as

$$\begin{aligned} \frac{d\mathcal{A}}{ds} &= \frac{1}{2m_b^8}\mathcal{B}_0 u(s)^2[8(Re\{(m_b^2 C_{BR} + m_b m_s C_{SL}^N)C_{10}^*) \\ &\quad +2s(\left|C_9^{eff} - C_{10}\right|^2 - \left|C_9^{eff} + C_{10}\right|^2)]. \end{aligned} \tag{2.12}$$

(The case for massive lepton with the all 12 operators is given in Appendix.) We note that, as is the case for the branching ratio, the FB asymmetry is more sensitive to the change of C_{BR} than of C_{SL} as shown in Figure 4, and the oscillating behavior reappears. To see the sensitivity of the asymmetry for each coefficient, we introduce the partially integrated (un)normalized FB asymmetry $\bar{\mathcal{A}}$ ($\mathcal{A}$) defined as

$$\begin{aligned} \bar{\mathcal{A}} &\equiv \frac{\mathcal{A}}{\mathcal{B}}, \\ \mathcal{A} &\equiv \int_1^8 ds \frac{d\mathcal{A}}{ds}, \end{aligned}$$

where $\mathcal{B} \equiv \int_1^8 ds \frac{d\mathcal{B}}{ds}$. We present the influence of two non-local coefficients on the normalized FB asymmetry in Figure 5. We note that, if we take into account only the C_{BR}

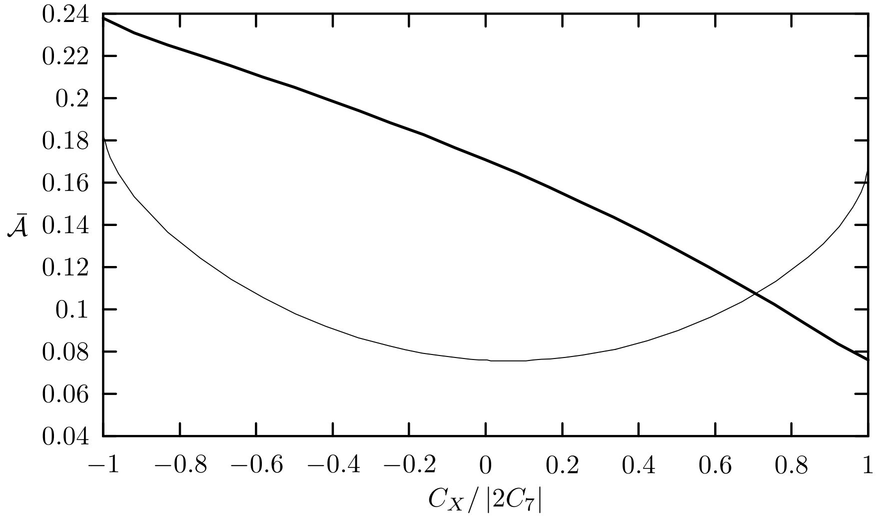

Figure 5: Partially integrated FB asymmetry $\bar{\mathcal{A}} \equiv \frac{\int_1^8 ds d\mathcal{A}/ds}{\int_1^8 ds d\mathcal{B}/ds}$ as the coefficients C_{SL}^N (thin line) and C_{BR} (thick line) change under Eqs. (2.8) and (2.9).

(as in the SM), or equivalently, if we assume $m_s = 0$, then the FB asymmetry decreases monotonously.

III Discussions and Conclusions

In Sec. II, we investigated both the branching ratio and the FB asymmetry independently. As shown, both observables are more sensitive to changes of C_{BR} than to those of C_{SL}, and oscillate as the non-local coefficients change. Now we show the correlation between the branching ratio and the FB asymmetry in Figure 6. It is very interesting to compare the correlation for various interactions, because the flows in the plane $(\mathcal{B}, \bar{\mathcal{A}})$ depend on interactions which we consider. We already investigated the correlation flows for the case of local interactions in [11]. The flows in the plain $(\mathcal{B}, \bar{\mathcal{A}})$ for the non-local interactions are quite different from the local ones. As found in Ref. [11], the standard model point is just near $(C_{SL}^N, C_{BR}) = (0, 2\,|C_7|)$ in the plane, so that it is placed at the lowest point (marked as $\diamond$) of the closed ellipse in Figure 6. Therefore, if there exist any non-local interactions in new theory beyond standard model, both the ratio and the FB asymmetry monotonically increase.

However, the vector-type interactions increase or decrease the branching ratio (the FB asymmetry) as the values of vector-type coefficients increase (decrease) or decrease (increase). And the scalar-type and tensor-type interactions make no change for the

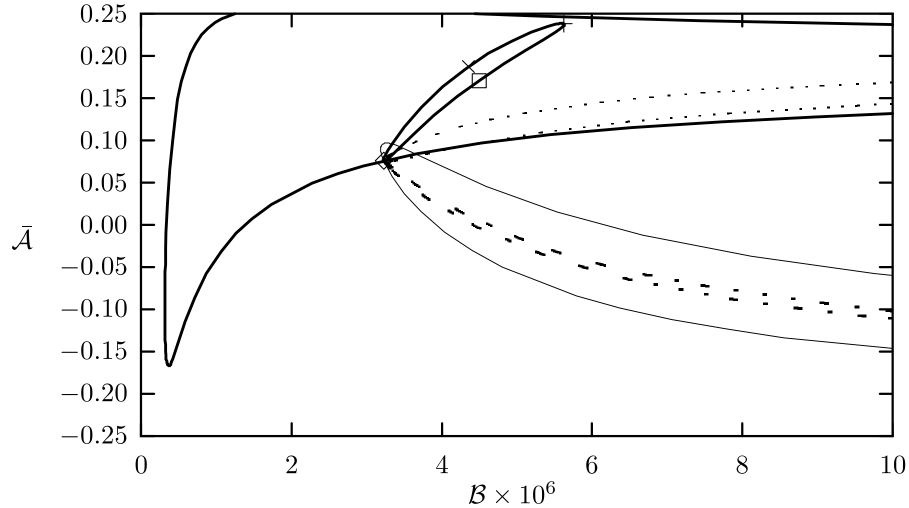

Figure 6: Correlation between the partially integrated branching ratio $\mathcal{B}$ and the partially integrated normalized forward-backward asymmetry $\bar{\mathcal{A}}$ for the non-local interactions (thick solid ellipse) and the vector-type interactions, C_{LL} (another thick solid line), C_{LR} (thin solid line), C_{RL} (thick dotted line) and C_{RR} (thin dotted line). The marks $\diamond$, $\square$, $+$ and $\times$ correspond to $(C_{SL}^N, C_{BR}) = (0, -2C_7)$, $(-2C_7, 0)$, $(0, 2C_7)$ and $(2C_7, 0)$, respectively. The point for the SM is placed just near the $(0, -2C_7)$.

unnormalized FB asymmetry. We can also understand the presented behavior for the non-local interactions with the following arguments: The branching ratio and the FB asymmetry change, for C_{BR} and C_{SL}, only through the second term in Eq. (2.10) and the first term in Eq. (2.12), since the two coefficients cannot change simultaneously under the condition (2.7). If we leave the leading term in m_s, the partially integrated branching ratio ($\mathcal{B}$) and the partially integrated forward-backward asymmetry ($\mathcal{A}$) are expressed as

$$\begin{aligned}
\mathcal{B} &= \mathcal{B}_{c1} + \mathcal{B}_{c2}[m_b C_{BR}(9 - 2m_b^2) + m_s C_{SL}^N (9 + 2m_b^2)] + \mathcal{O}(m_s^2), \quad (\mathcal{B}_{c2} > 0), \\
\mathcal{A} &= \mathcal{A}_{c1} - \mathcal{A}_{c2}(m_b C_{BR} + m_s C_{SL}^N) + \mathcal{O}(m_s^2), \quad (\mathcal{A}_{c2} > 0),
\end{aligned}$$

where $\mathcal{B}_{c1}$, $\mathcal{B}_{c2}$, $\mathcal{A}_{c1}$ and $\mathcal{A}_{c2}$ are independent of C_{BR} and C_{SL} and m_s. Under the condition $C_{SL} = \sqrt{4C_7^2 - C_{BR}^2}$, $\mathcal{B}$ and $\mathcal{A}$ achieve the minimum and the maximum at $C_{BR} = 2\,|C_7|$ and $-2\,|C_7|$. Therefore we will be able to know the sign of the C_7 and deviation from the predictions of the SM by using this correlation.

The non-local interactions are dominant as the momentum transfer from the b-quark to the s-quark gets comparable with the lepton mass, that is, as $\sqrt{s} \to m_l \sim 0$, because of the factor $1/s$. But, the sensitivity of the ratio to the changes of C_{BR} and C_{SL} is not so large in comparison with the local interactions, because of the constraint (2.7). If there is a new local interaction in addition, understanding the interference between the non-local

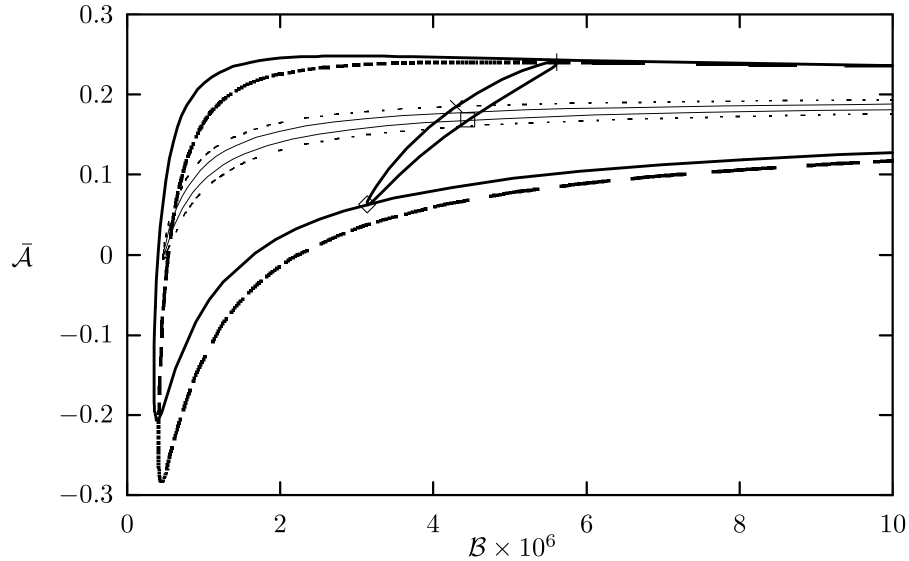

Figure 7: Correlation as the Wilson coefficients C_{LL} moves, receiving the effects of the interference between the new vector-type interaction and the new non-local interactions, whose Wilson coefficients are $(C_{SL}^N, C_{BR}) = (0, -2C_7)$ (thick solid line), $(0, 2C_7)$ (thick dashed line), $(-2C_7, 0)$ (thin solid line) and $(2C_7, 0)$ (thin dashed line). To refer, we also show the correlation as the set of the coefficients moves. The latter is also described in Figure 6 with the same notation.

interactions and the new local interaction would be extremely important within the small and non-vanishing s region, $1 < s < 8\ \ \mathrm{GeV}^2$ [26]. For example, in the massless limit, the scalar- and tensor-type interactions cannot contribute to the FB asymmetry. Hence, if we find a deviation of the FB asymmetry from the SM prediction, we can infer that there are new non-scalar- or non-tensor-type interactions. To extend further our discussion, suppose that interactions which act on massless leptons are equal to the ones which act on massive leptons. If we cannot find the lepton longitudinal polarization asymmetry $\left\langle P_L^+ \right\rangle + \left\langle P_L^- \right\rangle$ from the precise experiments for the decays $B \to X_s \tau^+ \tau^-$, then we can conclude there is no scalar- or tensor-type interaction [12], and infer that there are vector-type interactions like the SM and non-local interactions as well. In such a case, we should consider the interference between the non-local interactions and the vector-type ones. In Figure 7, we show the flow as C_{LL} moves when there are new non-local interactions, where, again, the branching ratio and the FB asymmetry are integrated over s from 1 GeV^2 to 8 GeV^2.

To summarize, we investigated the effects of the non-local interactions in the rare B decays $B \to X_s l^+ l^-$ in the model-independent way. In this analysis, we used all the operators which influence the process $B \to X_s l^+ l^-$, those are, ten local and two non-local four-Fermi operators. We studied the sensitivity to the coefficients of the non-local interactions for the branching ratio and the forward-backward (FB) asymmetry. We note that both the ratio and the FB asymmetry are more sensitive to C_{BR} than to C_{SL}. We did not use the C_{SL} introduced at first in Eq. (1.1), instead we used the normalized Wilson coefficient $C_{SL}^N \equiv \frac{m_s}{m_b} C_{SL}$, in order not to mislead. Nevertheless, the interference terms between the C_{SL}^N and the other local operators include an extra mass ratio m_s/m_b, compared to the interferences from the C_{BR} and others, and therefore, the operator C_{BR} gives greater influence on the ratio and the FB asymmetry than the C_{SL}. Consequently, the value of the C_{BR} almost decides the size of the branching ratio and the FB asymmetry as the result of their correlation. If there is any new charged local interaction, which contributes to $B \to X_s l^+ l^-$, like in the minimal supersymmetric standard model, we must consider appropriate non-local interactions, because any charged particle's interaction with photons yields non-local interactions. Especially, in the small invariant mass region, we cannot ignore the contribution from the non-local interactions. And our analysis would give very useful help for the precise study of new physics in $B \to X_s l^+ l^-$ when such a new local interaction exists.

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APPENDIX

A Branching Ratio and the Forward-backward Asymmetry with complete 12 Operators

In Ref. [11], we have already studied the differential branching ratio and the FB asymmetry for massless leptons without including two non-local operators, and in Ref. [12] we investigated the differential branching ratio and the polarization asymmetries for massive leptons with including all 12 operators. Here we show the differential branching ratio for massless leptons by including two new non-local interactions,

$$\begin{aligned}
\frac{d\mathcal{B}(s)}{ds} = \frac{1}{2m_b^8}\mathcal{B}_0 \quad [\quad & S_1(s)\{m_s^2|C_{SL}|^2 + m_b^2|C_{BR}|^2\} \\
& + S_2(s)\{2m_b m_s Re[C_{SL}C_{BR}^*]\} \\
& + S_3(s)\{2m_s^2 Re[C_{SL}(C_{LL}^* + C_{LR}^*)] + 2m_b m_s Re[C_{BR}(C_{RL}^* + C_{RR}^*)]\} \\
& + S_4(s)\{2m_b^2 Re[C_{BR}(C_{LL}^* + C_{LR}^*)] + 2m_b m_s Re[C_{SL}(C_{RL}^* + C_{RR}^*)]\} \\
& + S_5(s)\{2(m_s C_{SL} + m_b C_{BR})C_T^*\} \\
& + S_6(s)\{4(m_b C_{BR} - m_s C_{SL})C_{TE}^*\} \\
& + M_2(s)\{|C_{LL}|^2 + |C_{LR}|^2 + |C_{RL}|^2 + |C_{RR}|^2\} \\
& - M_6(s)\{2Re[C_{LL}C_{RL}^* + C_{LR}C_{RR}^*] \\
& \qquad - Re[C_{LRLR}C_{RLLR}^* + C_{LRRL}C_{RLRL}^*]\} \\
& + M_8(s)\{|C_{LRLR}|^2 + |C_{RLLR}|^2 + |C_{LRRL}|^2 + |C_{RLRL}|^2\} \\
& + M_9(s)\{16|C_T|^2 + 64|C_{TE}|^2\}.
\end{aligned} \tag{A.1}$$

The kinematic functions, $S_n(s)$ and $M_n(s)$, are all given in detail in Ref. [11, 12], and the behaviors of $S_n(s)(n = 1, 2, ..., 6)$ are shown in Figure 8. We find from Eq. (A.1) that $S_2(s)$, which includes m_s as an overall factor, is multiplied by m_b and m_s and, therefore, it is negligible. In Figure 8, where the lepton mass is set to that of muon ($\sim$ 105 MeV), we find that $S_2(s)$ are severely suppressed. We also show the most general form of the FB asymmetry in case of massive leptons, which includes all 12 operators. It is as follows:

$$\begin{aligned}
\frac{d\mathcal{A}}{ds} &= \frac{1}{2m_b^8}\mathcal{B}_0 u(s)^2[-4(Re\{m_b^2 C_{BR} + m_s^2 C_{SL})(C_{LL}^* - C_{LR}^*\}) \\
&\quad + 8m_b m_s Re\{(C_{BR} + C_{SL})(C_{RL}^* - C_{RR}^*)\} \\
&\quad + 4m_s m_l Re\{C_{SL}(C_{RLLR}^* + C_{RLRL}^*)\}
\end{aligned}$$

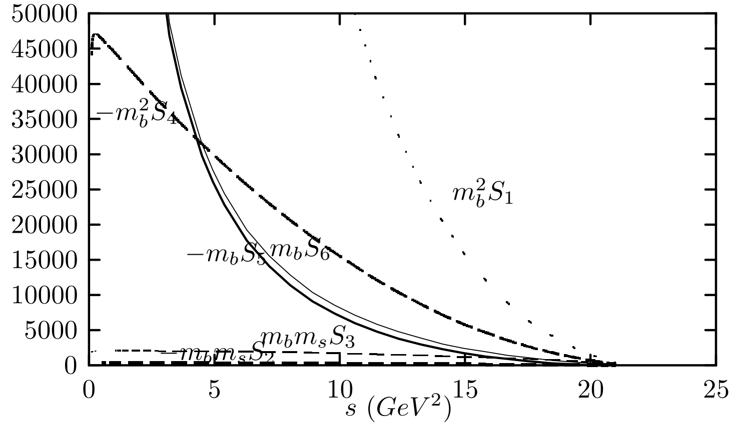

Figure 8: Behavior of the $S_n(s)(n = 1, 2, ..., 6)$. $m_b^2 S_1(s)$ (thin dotted line), $m_b m_s S_2(s)$ (thick dashed line), $m_b m_s S_3(s)$ (thin dashed line), $m_b^2 S_4(s)$ (dashed line), $m_b S_5(s)$ (thick solid line), and $S_6(s)$ (thin solid line). $S_2(s)$ are suppressed in comparison with the behavior of the other functions.

$$
\begin{aligned}
&+4m_b m_l Re\{C_{BR}(C^*_{LRLR} + C^*_{LRRL})\} \\
&+2s(|C_{LL}|^2 - |C_{LR}|^2 - |C_{RL}|^2 + |C_{RR}|^2) \\
&-8s(Re\{C_{LRLR}(C^*_T - 2C^*_{TE})\} + Re\{C_{RLRL}(C^*_T + 2C^*_{TE})\}) \\
&-2m_b m_l(Re\{(C_{LL} + C_{LR})(C^*_{LRLR} + C^*_{LRRL})\} \\
&\qquad\qquad + Re\{(C_{RL} + C_{RR})(C^*_{RLLR} + C^*_{RLRL})\}) \\
&-2m_s m_l(Re\{(C_{LL} + C_{LR})(C^*_{RLLR} + C^*_{RLRL})\} \\
&\qquad\qquad + Re\{(C_{RL} + C_{RR})(C^*_{LRLR} + C^*_{LRRL})\}) \\
&+24(m_b + m_s)m_l Re\{(C_{LL} - C_{LR} + C_{RL} - C_{RR})C^*_T\} \\
&+48(-m_b + m_s)m_l Re\{(C_{LL} - C_{LR} + C_{RL} - C_{RR})C^*_{TE}\}]. \qquad\qquad (A.2)
\end{aligned}
$$

If $C_{BR} = C_{SL} = -2C_7$, this corresponds with Eq. (A6) in Ref. [11].

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